Improvement of the renormalization method for the transition to stochastic instability in a Hamiltonian system and application to a harmonically forced double well potential

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(Received 7 July 1998; revised manuscript received 18 September 1998)

An improvement of the renormalization method for the Hamiltonian $H(v,x,t)=v^2/2-M\cos x-P\cos k(x-t)$ is pursued in the present paper. The coefficients of the retained resonances in the renormalization procedure are obtained using two different action values, one of which is taken from the renormalized torus and the other from the retained resonance centers. It turns out that the results for both cases show different behaviors at small k and M/P. The renormalization method is applied to the double well potential with an oscillatory force field which has not been explained by the two pendulum approximation. [S1063-651X(99)09403-9]

PACS number(s): 05.45.−a

I. INTRODUCTION

It is important in various fields to predict when a nonlinear Hamiltonian system shows a transition to a large-scale instability. Many authors have studied this subject for some specific systems by both direct numerical integration of the equation of motion and approximate theoretical methods [1,2].

Although many approximate criteria have been suggested for this transition, the most popular one among these is the so-called ''resonance overlap criterion'' [2]. This criterion holds that a transition to stochastic behavior occurs when the separatixes of two adjacent resonances start to overlap. However, a more accurate method was developed by Escande and Doveil using renormalization-group techniques [3,4]. This renormalization method was developed for a nonlinear Hamiltonian

$$H(v,x,t)=v^2/2-M\cos x-P\cos k(x-t), \tag{1}$$

and gave an estimate of the transition to stochastic instability within 5–10 % in comparison with the direct integration results. Escande and co-workers [5,6] pointed out that the Kolmogorov transformation gives more accurate estimates, and reported an estimate within 4%. Recently, Govin and co-workers [7,8] developed a more exact theory which can predict the stability and breakup of invariant tori in Hamiltonian flows, using a combination of Kolmogorov-Arnold-Moser theory and renormalization-group techniques.

The renormalization criterion given by Escande and co-workers was tested for several systems such as double well [9,10], periodic [11], and square well potential [12] systems with a monochromatic external field, and only the square well potential gave a very accurate prediction while the other cases did not. This bad prediction may come from the dependency of M and P on an action variable which does not allow a direct application of the renormalization method. In those systems, the application of the renormalization method should be carried out carefully.

In the present paper we manifest the improvement of the renormalization method by the Kolmogorov transformation, and apply the improved method to the double well potential with a monochromatic external field in a different way from the previous two pendulum approximation [9]. Our renormalization scheme basically follows that of Ref. [5]. The calculation in this paper, however, are performed in a different way. Coefficients of the resonances are approximately expressed in a closed form, and the two retained resonances are determined in a different way. The coefficients of the retained resonances are obtained using two different action values, one of which is taken from the renormalized torus and the other from the retained resonance centers, and the results of the both cases are compared.

In Sec. II the improved renormalization method by the Kolmogorov transformation is illustrated, and results of this impoved method are compared with the previous renormalization results and direct integration results [3] in Sec. III. The application to the double well with a monochromatic external field is discussed in Sec. IV, and we give conclusions in the final section.

II. RENORMALIZATION TRANSFORMATION

The Hamiltonian [Eq. (1)] can be transformed to a form which contains smaller oscillatory terms with second order of M and/or P by the Kolmogorov transformation. The Kolmogorov transformation from (v,x,t) to (I,θ,t) is carried out using a generating function

$$F(I,x,t)=Ix+\frac{M\sin x}{I}+\frac{P\sin k(x-t)}{k(I-1)} \tag{2}$$

which ''kills'' both resonances M and P. Then

$$v=\frac{\partial F}{\partial x}=I+\frac{M\cos x}{I}+\frac{P\cos k(x-t)}{I-1}, \tag{3}$$

$$\theta=\frac{\partial F}{\partial I}=x-\frac{M\sin x}{I^2}-\frac{P\sin k(x-t)}{k(I-1)^2}, \tag{4}$$

and the new Hamiltonian $H^{(1)}(I,\theta,t)$ is written as

$$H^{(1)}(I,\theta,t)=H_0(I)+V(I,\theta,t), \tag{5}$$

where

$$H_0(I)=\frac{1}{2}I^2+\frac{M^2}{4I^2}+\frac{P^2}{4(I-1)^2}, \tag{6}$$

$$V(I,\theta,t)=\frac{M^2}{4I^2}\cos 2x+\frac{P^2}{4(I-1)^2}\cos 2k(x-t)+\frac{MP}{I(I-1)}\cos x\cos k(x-t). \tag{7}$$

Note that $V(I,\theta,t)$ is of second order of M and/or P. We assume that k is a rational number, i.e., $k=r/p$, where r and p are integers. The perturbative term $V(I,\theta,t)$ is, then, a periodic function for θ and t with a period $2\pi p$, and can be expanded into the Fourier series:

$$H(I,\theta,t)=H_0(I)+\sum_{n,l} V_{nl}(I)\cos((nk+l)\theta-nkt). \tag{8}$$

The Fourier coefficient $V_{nl}(I)$ can be calculated approximately. The complex form of $V(I,\theta,t)$ is

$$U(I,\theta,t)=\frac{M^2}{4I^2}e^{i2x}+\frac{P^2}{4(I-1)^2}e^{i2k(x-t)}+\frac{MP}{2I(I-1)}(e^{i((1+k)x-kt)}+e^{i((1-k)x+kt)}), \tag{9}$$

and its Fourier coefficients are

$$U_{jm}=\frac{1}{\tau^2}\int_0^\tau\int_0^\tau U(I,\theta,t)e^{-(2\pi/\tau)(j\theta+mt)}d\theta dt, \tag{10}$$

where $\tau=2\pi p$. In order to obtain the analytic expression, we approximate the expression of x as

$$x\simeq\theta+\frac{M\sin\theta}{I^2}+\frac{P\sin k(\theta-t)}{k(I-1)^2}, \tag{11}$$

and use the identity relation

$$e^{iz\sin\phi}=J_0(z)+\sum_{n=1}^{\infty}J_n(z)(e^{in\phi}+(-1)^n e^{-in\phi}), \tag{12}$$

where the J's are Bessel functions. Substituting Eqs. (11) and (12) into Eq. (9), one can see that U_{jm} has nonvanishing values only when $j=nr+lp$ and $m=nr$, where n and l are integers, and can obtain an expression for U_{jm}, from which, in turn, V_{nl} should be obtained. Several values of V_{nl} for a typical I value are shown in Fig. 1. It is shown that the dominant coefficients are values belonging to $l=1$ in the region concerned. Therefore, hereafter we suppress the index l by putting $l=1$. In Ref. [5] the Fourier coefficients are expressed by a infinite series, and the dominant coefficients are chosen by putting $n=1$ instead of $l=1$.

The position of the resonance characterized by n and $l=1$ in the angular velocity Ω is found by the condition that the phase of the resonance be stationary. The position of the resonance is

$$\Omega_{(n)}=\frac{nk}{nk+1}. \tag{13}$$

Taking an invariant torus characterized by I_0, the above Hamiltonian can be expanded about I_0 or the corresponding angular velocity Ω_0. Here we assume that the invariant torus is affected by the neareast two resonances that are at $\Omega_{(n_0)}$ and $\Omega_{(n_0+1)}$ and, then,

$$\Omega_{(n_0)}<\Omega_0<\Omega_{(n_0+1)}. \tag{14}$$

We assume $n_0\geqslant 1$, and the case $n_0=0$ (the invariant tori between resonances $\Omega_{(0)}$ and $\Omega_{(1)}$) will be discussed at the end of this section. We denote the position of the invariant torus by a noninteger value z, i.e.,

$$\Omega_0=\Omega_{(z)}=\Omega_{(n_0+\delta z)},\quad 0\leqslant\delta z<1. \tag{15}$$

Then the effective Hamiltonian becomes

$$H^{(1)}_{eff}(I,\theta,t)=H_0(I_0)+\Omega_0(I-I_0)+\tfrac{1}{2}\sigma_0(I-I_0)^2+V_{n_0}(I_0)\cos((n_0k+1)\theta-n_0kt)+V_{n_0+1}(I_0)\cos[((n_0+1)k+1)\theta-(n_0+1)kt], \tag{16}$$

where $\Omega_0=dH_0(I_0)/dI$ and $\sigma_0=d^2H_0(I_0)/dI^2$.

In order to obtain the desired Hamiltonian, a canonical transformation is carried out. Using the following generating function, the effective Hamiltonian is transformed from (θ,I,t) to (y,J,t):

$$F^{(1)}(I,y,t)=-\frac{I(y+(n_0+\lambda)kt)}{(n_0+\lambda)k+1}+\mu y+\nu t, \tag{17}$$

where

$$\mu=\frac{(n_0+\lambda)k-(\Omega_0-\sigma_0 I_0)((n_0+\lambda)k+1)}{\sigma_0((n_0+\lambda)k+1)^2} \tag{18}$$

and

$$\nu=-H_0+\Omega_0 I_0-\frac{1}{2}\sigma_0 I_0^2 +\left(\sigma_0 I_0-\Omega_0+\frac{(n_0+\lambda)k}{(n_0+\lambda)k+1}\right)((n_0+\lambda)k+1)\mu -\frac{1}{2}\sigma_0((n_0+\lambda)k+1)^2\mu^2, \tag{19}$$

where λ is introduced to make renormalization procedures have fast convergence [3], and

$$\lambda=0 \quad \text{for} \quad \tfrac{1}{2}\leqslant\delta z<1, \qquad \lambda=1 \quad \text{for} \quad 0\leqslant\delta z<\tfrac{1}{2}. \tag{20}$$

This choice of λ guarantees that $n_0\geqslant 1$ at successive renormalization procedures, which will be clear in Sec. III. From the above generating function, the relations between the old and new coordinates are

$$J=-\frac{\partial F^{(1)}}{\partial y}=\frac{I}{(n_0+\lambda)k+1}-\mu, \tag{21}$$

$$\theta=-\frac{\partial F^{(1)}}{\partial I}=\frac{y+(n_0+\lambda)kt}{(n_0+\lambda)k+1}, \tag{22}$$

and the new Hamiltonian is obtained as

$$H^{(2)}=H_{\rm eff}^{(1)}+\frac{\partial F^{(1)}}{\partial t} =\frac{1}{2}\sigma_0((n_0+\lambda)k+1)^2 J^2+V_{n_0+\lambda}\cos y +V_{n_0+1-\lambda}\cos k'(y-\gamma t), \tag{23}$$

where

$$k'=\frac{(n_0+1-\lambda)k+1}{(n_0+\lambda)k+1} \tag{24}$$

and

$$\gamma=\frac{(1-2\lambda)k}{(n_0+1-\lambda)k+1}. \tag{25}$$

At last, we have the final Hamiltonian with $t'=\gamma t$ and $w=[\sigma_0((n_0+\lambda)k+1)^2/\gamma]J$,

$$H^F(w,y,t')=\frac{w^2}{2}-M'\cos y-P'\cos k'(y-t'), \tag{26}$$

where

$$M'=-\frac{\sigma_0((n_0+\lambda)k+1)^2}{\gamma^2}V_{n_0+\lambda}, \tag{27}$$

$$P'=-\frac{\sigma_0((n_0+\lambda)k+1)^2}{\gamma^2}V_{n_0+1-\lambda}. \tag{28}$$

A renormalization procedure is composed of the transformations illustrated in this section, i.e., it corresponds to a transformation from Hamiltonian parameters (k,M,P) to (k',M',P').

Until now we have assumed that $n_0\geqslant 1$, which means that the invariant torus to be renormalized exists in one part of a possible region between resonances M and P, i.e., $\Omega_{(1)}<\Omega_0<1$. The renormalization procedure for the other part $(\Omega_{(0)}<\Omega_0<\Omega_{(1)})$ can be performed with the help of a transformation from the original Hamiltonian [Eq. (1)], with parameters (k,M,P) to an identical Hamiltonian system with parameters $(1/k,P,M)$. Then the range $\Omega_{(0)}<\Omega_0<\Omega_{(1)}$ in the original Hamiltonian corresponds the range $\Omega_{(1)}<\Omega_0<1$ in the transformed Hamiltonian with parameters $(1/k,P,M)$, so we can again apply, the above renormalization procedure to this Hamiltonian.

III. NUMERICAL RESULT OF RENORMALIZATION TRANSFORMATION

As mentioned in Sec. II, the invariant torus to be renormalized can be characterized by z or n_0 and δz. In order to carry out successive renormalizaion procedures, the position of transformed invariant torus $(z'=n_0'+\delta z')$ should be required. This position can be easily obtained when one sees the problem in the limit that M and P go to zero. From Eq. (13), the position of invariant torus can be written as

$$z=n_0+\delta z=\frac{\Omega_0}{k(1-\Omega_0)}, \tag{29}$$

and, in the limit that M and P go to zero,

$$z=\frac{v_0}{k(1-v_0)}, \tag{30}$$

since, in that limit, $v=I=\Omega$. The position of transformed invariant torus, therfore, is

$$z'=n_0'+\delta z'=\frac{w_0}{k'(1-w_0)}, \tag{31}$$

and, using Eq. (21) and the relation between w and J in Sec. II, one can obtain

$$z'=\frac{\delta z}{1-\delta z} \quad \text{for} \quad \lambda=0 \tag{32}$$

and

$$z'=\frac{1-\delta z}{\delta z} \quad \text{for} \quad \lambda=1. \tag{33}$$

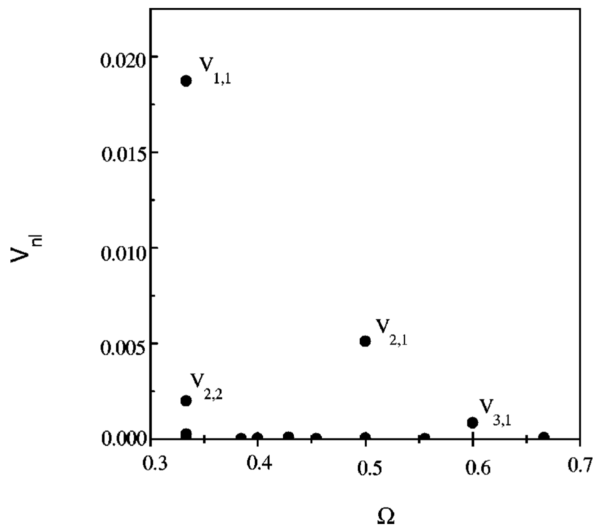

FIG. 1. V_{nl} when $M=0.1$, $P=0.1$, $k=0.5$, and $I=0.2$.

Note that the new position of the invariant torus depends only on δz, not on n_0. The relation between z' and δz, and the fact $n_0'\geqslant 1$ (the integer part of z' is n_0'), are shown in Fig. 2. There is the infinite number of fixed points δz_n^λ as shown in Fig. 3. Those fixed points are

$$\delta z_n^{\lambda=0}=-\frac{n}{2}+\frac{n}{2}\sqrt{1+\frac{4}{n}}, \tag{34}$$

$$\delta z_n^{\lambda=1}=\frac{-n-1}{2}+\frac{1}{2}\sqrt{n^2+2n+5}, \tag{35}$$

where n is a positive integer. This means that if one takes an invariant torus corresponding to $z=n_0+\delta z_{n_0'}^\lambda$, then the next position of invariant torus is $z'=n_0'+\delta z_{n_0'}^\lambda$ and this position is unchanged under subsquent renormalization procedures.

As one can see from Eq. (24), under successive application of renormalization procedures the value of k approaches

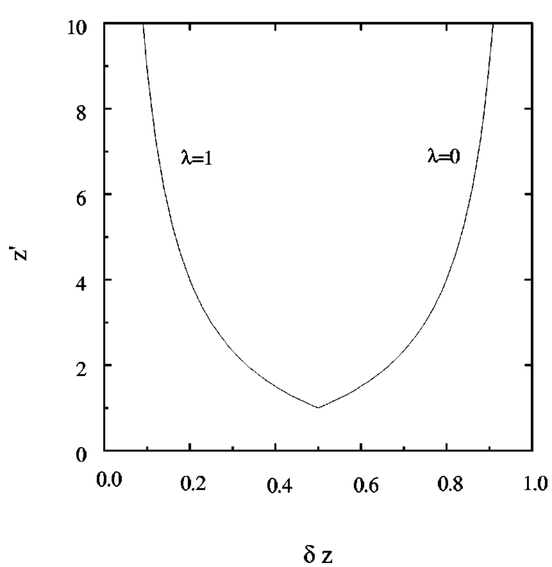

FIG. 2. z' as a function of δz.

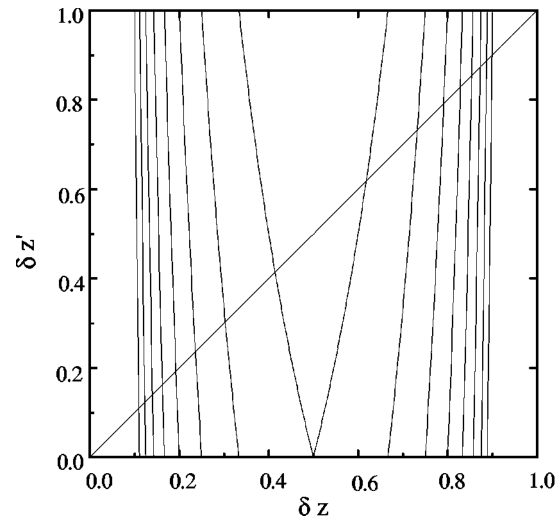

FIG. 3. $\delta z'$ vs δz. The cross points with $\delta z'=\delta z$ line are the fixed points.

two fixed points, one of which corresponds to $\lambda=0$ and the other to $\lambda=1$ (Fig. 4). These fixed points are

$$k_n^{\lambda=0}=\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4}{n}}, \tag{36}$$

$$k_n^{\lambda=1}=\frac{1}{2}-\frac{1}{n+1}+\frac{1}{2}\sqrt{1+\frac{4}{(n+1)^2}}. \tag{37}$$

Once one takes an invariant torus, one can know whether the invariant torus is broken or not from the behavior of the transformed coefficients M and P under successive renormalization. When the values of the transformed M and P approach zero, the invariant torus is stable, i.e., not broken. On the other hand, if the values become larger and larger successively, the invariant torus is supposed to be broken.

In practical calculations, we use two kinds of coefficients of the retained resonances in Eq. (23). One is obtained by taking the action value of the renormalized torus, and the other by taking the action value of resonance center. (Results

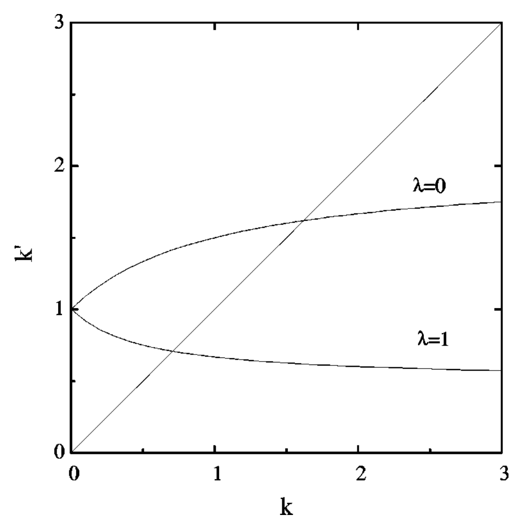

FIG. 4. k' vs k when $n=1$. The two cross points with the $k'=k$ line are the fixed points.

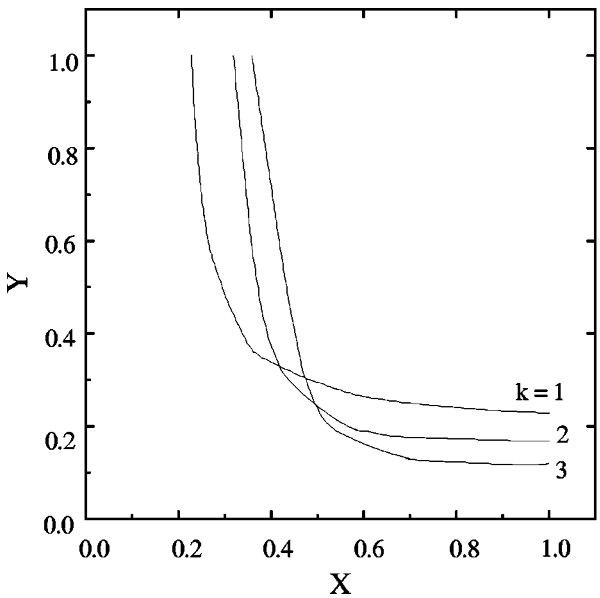

FIG. 5. The instability transition lines when $k=1$, 2, and 3.

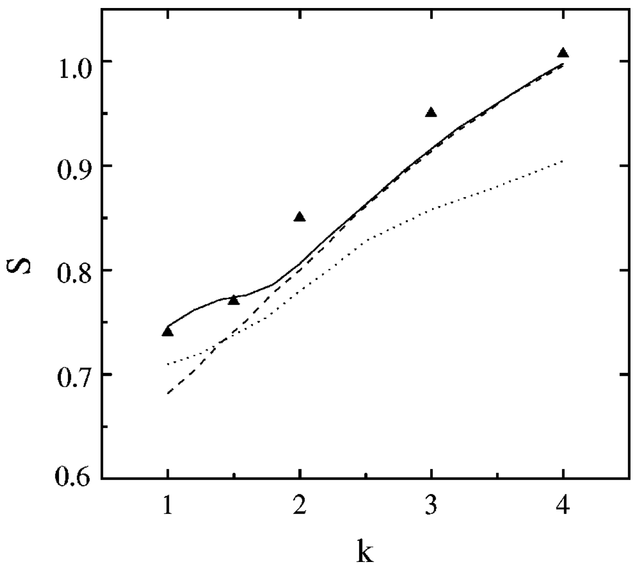

FIG. 7. S vs k when $X/Y=1$. The solid line and dashed lines represent the present results for the cases of the use of the action value of the renormalized torus and of the resonance center, respectively. The dotted line and the data points are results of Ref. [3].

for the latter case appear only in Figs. 6 and 7) We investigate the stability about invariant tori $z=n_0+\delta z^{\lambda}_{n_0'}$ and $1\leqslant n_0, n_0' \leqslant 10$. This means that 400 invariant tori, being between resonances M and P, are investigated. This number of invariant tori is enough to see the transition to stochastic instability from the isolated resonances divided by a stable invariant torus. We can suppose that when the invariant torus is broken, a transition to stochastic instability takes place.

Results for $k=1$, 2, and 3 are shown in Fig. 5, where $X=2\sqrt{M}$ and $Y=2\sqrt{P}$ indicate the half width of the resonant domains. These results are very similar to those of the previous the renormalization method [3].

To see the improvement of the renormalization method by the Kolmogorov transformation, in Figs. 6 and 7 we plot S versus X/Y when $k=1$, and S versus k when $X/Y=1$, respectively, where $S=X+Y$. The solid and dashed lines represent the present results for the cases of the use of the action value of the renormalized torus and the retained resonance center, respectively. The dotted line shows the results of the previous renormalization method and the data points those of the direct numerical integration [3].

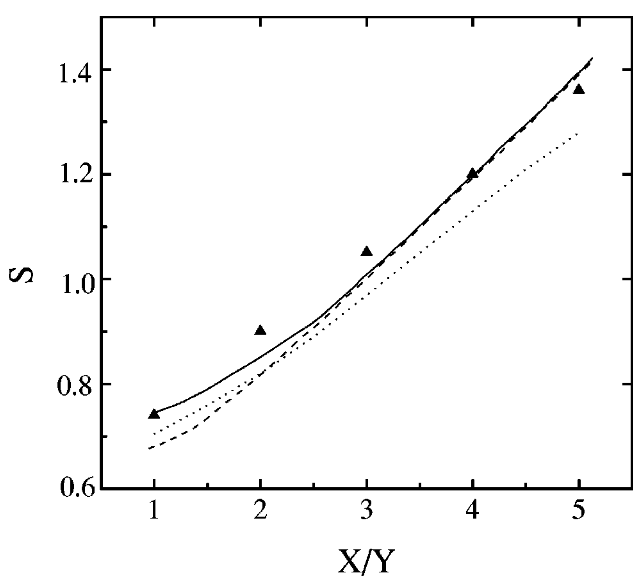

FIG. 6. S vs X/Y when $k=1$. The solid line and dashed lines represent the present results for the cases of the use of the action value of the renormalized torus and of the resonance center, respectively. The dotted line and the data points are results of Ref. [3].

It is clear from the figures that the Kolmogorov transformation improves the renormalization method when using the action value of the renormalized torus. When using the action value of the resonance center, the estimates of the transition to stochastic instability do not improved at small X/Y and k. These results are different from the known fact that the use of the action value of the resonance center is crucial in the improved renormalization scheme by the Kolmogorov transformation [5]. We find that the behavior at small X/Y and K is determined by invariant tori with small n_0, where the distance between the retained resonances is relatively long. This implies that the use of the action value of the retained resonance center is not proper when the distance is long.

IV. APPLICATION TO THE DOUBLE WELL WITH AN OSCILLATORY FORCE FIELD

Reichl and Zheng [9,10] studied the motion of a particle trapped in a quartic double-well potential in the presence of a dynamic monochromatic external field. They showed that the use of the renormalization method does not give a proper result in this system under their pendulum approximation. We note that their pendulum approximation is somewhat unphysical in choosing the coefficients of perturbations corresponding to M and P in Eq. (1). In this section, we apply the renormalization method to this system in a subtle way.

The Hamiltonian for this system is

$$H=\tfrac{1}{4}p^2-2x^2+x^4+\epsilon x\cos(\omega t), \tag{38}$$

and can be rewritten in terms of action-angle variables [10] as

$$H=E_0(I)+\epsilon\sum_{n=-\infty}^{\infty} g_n(I)\cos(n\theta-\omega t), \tag{39}$$

where

$$g_n(I)=\frac{\pi}{2K}\sqrt{\frac{2}{2-k^2}}\,\mathrm{sech}\left(\frac{|n|\pi K'}{K}\right) \tag{40}$$

for a trapped particle $(-1<E_0<0)$.K is the complete elliptic integral of the first kind, k is the modulus defined as

$$k^2=\frac{2\sqrt{1+E_0}}{1+\sqrt{1+E_0}}, \tag{41}$$

and K' is the complete elliptic integral of the first kind with a modulus $k'=\sqrt{1-k^2}$. The position of the nth resonance zone in action space is

$$\dot{\theta}\simeq\left.\frac{\partial E_0}{\partial I}\right|_{I=I_n}=\left.\frac{\sqrt{2}\pi}{\sqrt{2-k^2}K}\right|_{I=I_n}=\frac{\omega}{n}. \tag{42}$$

In order to study the onset of chaos in the region between the nth and $(n+1)$st zone, one can use the two resonance approximation and expand about the resonance point I_n,

$$\begin{aligned}H=&E_0(I_n)+\frac{\partial E_0(I_n)}{\partial I}(I-I_n)+\frac{1}{2}\frac{\partial^2 E_0(I_n)}{\partial I^2}(I-I_n)^2+\cdots\\&+\epsilon g_n(I)\cos(n\theta-\omega t)\\&+\epsilon g_{n+1}(I)\cos[(n+1)\theta-\omega t].\end{aligned} \tag{43}$$

In the pendulum approximation given by Reichl and Zheng, the coefficients are taken as $g_n(I)=g_n(I_n)$ and $g_{n+1}(I)=g_{n+1}(I_{n+1})$. However, these replacements have a weak physical basis, since $g_n(I)$ and $g_{n+1}(I)$ are the functions of the same action I.

Now we concentrate on the case of $n=1$. Using the generating function

$$F_1(I,x',t)=-(I-I_1)(x'+\omega t), \tag{44}$$

a new Hamiltonian can be obtained as

$$\begin{aligned}H_1'=H+\frac{\partial F_1}{\partial t}=&E_0(I_1)-\frac{1}{2}\frac{p'^2}{m_0(I_1)}+\epsilon g_1(I)\cos x'\\&+\epsilon g_2(I)\cos(2x'+\omega t),\end{aligned} \tag{45}$$

where

$$m_0(I_1)=\left|\left(\frac{\partial^2 E_0(I_1)}{\partial I^2}\right)\right| \tag{46}$$

and

$$\frac{\partial^2 E_0(I)}{\partial I^2}=\frac{\pi^2(2-k^2)}{4k^4K^3}\left(2K-\left(\frac{2-k^2}{1-k^2}\right)E\right), \tag{47}$$

where E is the complete elliptic integral of the second kind.

Since we have interest in the onset of chaos in the region between $n=1$ and 2 zones, it is possible to expand the Hamiltonian about the $n=2$ resonance; then we obtain a Hamiltonian similar to Eq. (43). Then, if we use the generating function

$$F_2(I,x',t)=-\tfrac{1}{2}(I-I_2)(x'+\omega t), \tag{48}$$

a new Hamiltonian is found as

$$\begin{aligned}H_2'=H+\frac{\partial F_2}{\partial t}=&E_0(I_2)-2\frac{p'^2}{m_0(I_2)}+\epsilon g_1(I)\cos\frac{1}{2}(x'-\omega t)\\&+\epsilon g_2(I)\cos x',\end{aligned} \tag{49}$$

where

$$m_0(I_2)=\left|\left(\frac{\partial^2 E_0(I_2)}{\partial I^2}\right)\right|. \tag{50}$$

If one takes $x''=\frac{1}{2}(x'-\omega t)$ and $p''=2p-\frac{1}{2}\omega m_0(I_2)$, it is evident that this Hamiltonian is identical to H_1' except for the value of m_0. So it is possible that the error inserted through the expansion can be reduced by taking the average value of $\overline{m_0}=(m_0(I_1)+m_0(I_2))/2$.

Taking $p_0=2p'/\omega$, $t_0=\omega t/2$, $x_0=x'$, and $H_0=4H'/\omega^2$, the Hamiltonian H_1' takes the so-called "standard form"

$$H_0=\frac{4E_0(I_1)}{\omega^2}-\frac{p_0^2}{2\overline{m_0}}+U_0^x\cos x_0+U_0^y\cos(2(x_0+t_0)). \tag{51}$$

The coefficients are defined as

$$U_0^x=\frac{4\epsilon g_1(I)}{\omega^2} \tag{52}$$

and

$$U_0^y=\frac{4\epsilon g_2(I)}{\omega^2}. \tag{53}$$

These are related to our coefficients M and P by

$$M=\frac{U_0^x}{\overline{m_0}} \tag{54}$$

and

$$P=\frac{U_0^y}{\overline{m_0}}. \tag{55}$$

In order to apply the renormalization method, it is necessary to set the values of $g_1(I)$ and $g_2(I)$. Since the invariant tori $z=n_0+\delta z_{n_0'}^{\lambda}$ are investigated (see Sec. III), it is natural to take $g_1(I_z)$ and $g_2(I_z)$, where I_z is the corresponding action for the renormalized torus. This means that we use different values of the coefficients according to the position of invari-

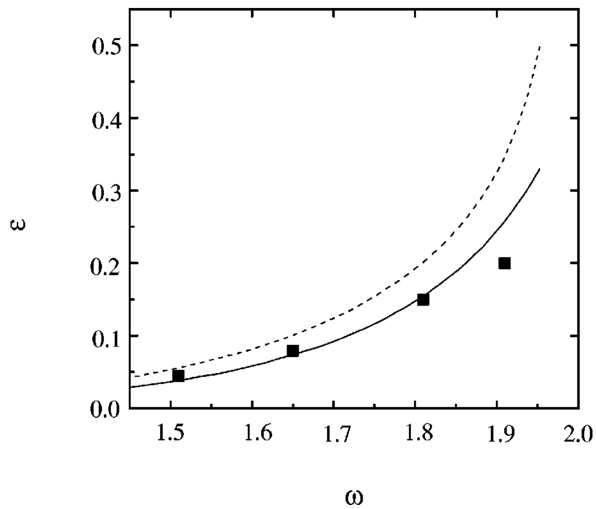

FIG. 8. The instability transition line on the ϵ-ω plane. The solid line represents the present theory. The dashed line shows the result of the two pendulum approximation. The data points show direct integration [9].

ant torus. The result is shown in Fig. 8. Our careful application of the renormalization gives a more accurate estimate than the two pendulum approximation.

V. CONCLUSION

We confirm the improvement of the renormalization method by the Kolmogorov transformation for the transition to stochastic instability in a Hamiltonian [Eq. (1]. As pointed out in Refs. [3,5], this improvement can be expected, since the Kolmogorov transformation makes the perturbative terms smaller in the second order of M and P in the original Hamiltonian, so that the expansion of Hamiltonian about I_0 in Sec. II gives a more accurate expression than does the expansion without the Kolmogorov transformation. However, in the present renormalization scheme the use of the action value of the resonance center for the coefficients gives a rather bad estimate at small k and X/Y, which is contrary to the results of Ref. [5]. The origin of this discrepancy is not clear. It seems that the use of the action value of the retained resonance center is not proper when the distance between the resonance centers is long. However, we suppose that the use of the action value of the resonance center does not give good estimates when the retained resonances locate with a distance, i.e., when n_0 is small.

Using this renormalization method, we also investigate the escape of a particle trapped in a double well system due to a dynamic monochromatic external field. In this application, varying coefficients of the pertubation according to the invariant torus tested are used, which have a more solid physical basis than the previous two pendulum approximation of Reichl and Zheng.

ACKNOWLEDGMENT

One of the authors (E.S.Y.) acknowledges the partial support by the research fund of Semyung University.

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